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3P Sampling

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ACKNOWLEDGMENTS

Appreciation for suggestions is expressed to Joseph E. Barnard, H. Gyde Lund, and James C. Space, USDA Forest Service, and Dr. George M. Furnival, Yale University.

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PREFACE

The 3P sampling design, the "3P" indicating probability proportional to prediction, is very efficient for many applications in forest inventory. L. R. Grosenbaugh was responsible for developing the sampling system and included it in very comprehensive and complicated computerized inventory programs utilizing sophisticated and expensive dendrometers. However, the basic ideas of 3P sampling are simple, and useful applications are possible without unusual computer or dendrometry equipment. It is to this end that this publication is dedicated.

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Elementary 3P Sampling

HARRY V. WIANT, JR.

INTRODUCTION

There has been an interesting evolution in forest sampling designs. It might be illustrated as follows:

100% tally → fixed-area plots → point sampling → 3P sampling

The measurement of each tree, which is still common in many parts of the world and in many timber sales in this country, will provide, other things being equal, the most accurate estimation of total volume. As each tree in the population of interest is measured, there is no sampling error, in a statistical sense. The disadvantages of this procedure, with the labor costs of today, are obvious. For inventories of thousands of trees on hundreds of acres, measuring each tree is, for all practical purposes, impossible.

Fixed-area samples, such as 1/5-acre plots or chain-wide strips, were introduced for selecting trees to measure. Although the theoretical advantages of randomly locating these samples are recognized, they are usually established on a systematic pattern. The frequency of measuring trees of given sizes on fixed-area plots depends on their frequency of occurrence. As a result, at the conclusion of a cruise it is often evident that too many small, low-value trees and too few large, high-value trees were included in the sample.

As an attempt to better distribute the sample among size classes, the point sampling system was developed. This scheme selects sample trees with probability proportional to their basal areas, providing a much more efficient sample than methods selecting samples with probability proportional to frequency of occurrence. There is a strong relationship between basal area and volume. However, that relationship is not perfect. For example, it is not difficult to visualize two trees with identical basal areas but with very different sawtimber volumes, due to differences in taper, height, branching habits, etc.

It would be an obvious advantage to select sample trees with probability proportional to their volumes or values. This is what the 3P system is designed to do. Actually, as we shall see, the 3P system is usually designed to select sample trees using guessed volumes or values. To understand the efficiency of this approach, let us review a little sampling theory.

SAMPLING THEORY

Most forests, especially natural forests, have large variations between volumes of individual trees or total volumes of trees on given size plots. This variation is conveniently expressed as the coefficient of variation, which relates the standard deviation to the mean. The formula is:

$$CV(\%) = \frac{S}{\bar{x}}(100) \qquad (1)$$

where: CV = coefficient of variation expressed as a percent

S = standard deviation

$\bar{x}$ = mean

The mean ($\bar{x}$) is simply the sum of the observations divided by the number of observations made. The standard deviation provides a measure of dispersion with which to qualify the mean of a normally distributed population, a population yielding a bell-shaped curve peaking at the mean. In such a population, about 68 percent of the individual values would be expected to fall within the values of the mean plus or minus one standard deviation; the mean plus or minus two standard deviations should include approximately 95 percent of the population. The standard deviation is calculated as:

$$S = \sqrt{\frac{\Sigma X^2 - (\Sigma X)^2/n}{n - 1}} \qquad (2)$$

where: S = standard deviation expressed in the same units as is the mean

Σ = sum

X = an individual observation

n = number of observations

For example, with four observations of 10, 5, 15, and 10, which total 40 and average 10:

$$S = \sqrt{\frac{(10)^2 + (5)^2 + (15)^2 + (10)^2 - (40)^2/4}{4 - 1}}$$

$$= 4.1$$

$$CV\% = \frac{4.1}{10}(100) = 41\%$$

The coefficient of variation for the volumes of individual trees in West Virginia forests, for example, approximates 100 percent. Also, plot volumes of fixed-area or point samples have coefficients of variation of about 100 percent in this area.

A useful formula for calculating the approximate number of random samples required for a specified sampling error is:

$$n = \frac{t^2 CV^2}{E^2} \tag{3}$$

where: n = number of samples required

t = t-value; for most inventory work, 1 is used which should provide the desired sampling error 2 times in 3

CV = coefficient of variation as a percent (e.g., 10% = 10)

E = sampling error acceptable as a percent (e.g., 5% = 5)

We now have the tools necessary to examine the efficiency of 3P sampling. If instead of measuring all trees on a timber marking job in which all trees are visited, we decide to measure trees selected on a random basis and keep only a count of non-sample trees, the number of sample trees needed for a total volume estimate within a sampling error of 2 percent, 2 in 3 times, for a coefficient of variation of 100 percent is:

$$n = \frac{(1)^2(100)^2}{(2)^2}$$

$$= 2500$$

As we will see later, the variation in 3P sampling is related to the ratio of measured to guessed volumes, and most workers report coefficients of variation of about 20 percent. Instead of the 2500 sample trees required in the example above, 3P sampling would require visiting every tree for guessed volumes and the measurement of:

$$n = \frac{(1)^2(20)^2}{(2)^2}$$

$$= 100 \text{ trees}$$

For the same accuracy in our total volume figure, 3P sampling in this example requires the measurement of 1/25th as many sample trees. We can therefore afford to measure these trees with greater accuracy than might have been possible with a simple random sample. Before studying 3P sampling in greater depth, we shall examine the scheme on which 3P is based, list sampling.

LIST SAMPLING

If one has a list of values for the population of interest, samples can be drawn with probability proportional to the values on the list. One advantage list sampling has over the closely related 3P system is that with list sampling one is assured of obtaining the exact number of samples desired. As we shall see, this is not possible with 3P sampling.

For example, let us suppose that the numbers in column B of Table 1 represent the individual scaled volumes in cubic feet of a population of ten logs.

Column C is a cumulative total of column B. Column D gives numbers from which random samples will be drawn. It can be seen that the second assigned number, or only number if a value in column B equals 1, equals the cumulative total in that row. The first assigned number in a row equals the second assigned number in the previous row plus 1. Random numbers in this case would be drawn from 1 through 34. If random numbers 5, 16, and 31 are drawn, item numbers 3, 5, and 10, respectively, will be measured. The probability of a sample being selected is proportional to its size. In theory, sampling is with replacement, which means an item designated in column A may appear in the sample more than once. In practice, with large populations and relatively small sample sizes, the difference between sampling with or without replacement can be ignored (Loetsch *et al.* 1973).

Table 1. Data for list sampling examples.

A Item No.	B Volume (X)	C Cumulative totals	D Assigned range	E Volume (Y)	F Y/X
1	1	1	1		
2	3	4	2-4		
3	2	6	5-6	4	2.0000
4	4	10	7-10		
5	10	20	11-20	12	1.2000
6	3	23	21-23		
7	2	25	24-25		
8	1	26	26		
9	2	28	27-28		
10	6	34	29-34	7	1.1667
	Total 34			Total	4.3667
				Average	1.45557

Column E gives the measured volume of sample logs. We are now ready to calculate quantities of interest.

The total volume estimate for the ten logs is:

$$\begin{aligned}\text{Total vol.} &= (\text{the average Y/X ratio})\ (\text{column B total}) \qquad (4)\\ &= (1.45557)\ (34)\\ &= 49\end{aligned}$$

Using the procedure indicated by Mesavage (1971) and Space (1973), the sampling error, which is the standard error expressed as a percent of the mean, for the population of ten logs is calculated as:

$$\text{Sampling error (\%)} = \frac{\text{CV for ratios}}{\sqrt{\text{number of list (or 3P) samples}}} \qquad (5)$$

where:

$$\text{CV for ratios} = \frac{\text{S for ratios}}{\text{average ratio}}\ (100) \qquad (6)$$

and, as shown in formula 2,

$$S = \sqrt{\frac{(2.0000)^2 + (1.2000)^2 + (1.1667)^2 - \left[\frac{(4.3667)^2}{3}\right]}{3 - 1}}$$

$$= 0.47179$$

Therefore, CV as calculated by formula 1, is:

$$\begin{aligned}CV &= \frac{0.47179}{1.45557}\ (100)\\ &= 32.4\%\end{aligned}$$

Sampling error, then, is:

$$\begin{aligned}\text{Sampling error} &= \frac{32.4}{\sqrt{3}}\\ &= 18.7\%\end{aligned}$$

Rather than considering the ten numbers in column B of Table 1 as representing a population of logs, let us assume they represent the per-acre volumes (M.B.F.) of ten randomly located CFI plots measured in 1970. To update volume estimates in 1975 without measuring all plots again, list sampling is used drawing the same samples as in the previous example.

Per-acre per-year growth for CFI plots is:

$$\text{Growth} = \frac{(1975 \text{ volume}) - (1970 \text{ volume})}{(\text{years between measurements})\ (\text{number of CFI plots})} \qquad (7)$$

$$= \frac{49 - 34}{(5)\ (10)}$$

$$= 0.30 \text{ M.B.F.}$$

The sampling error calculation for the ten CFI plots is more complicated, for the plots are samples of the forest, with their own sampling error, and there is a sampling error due to the list sample. In addition, a covariance term should be included (see Wiant 1975a), but will be ignored in this bulletin as is usually done in practice (Van Hooser 1972, 1973).

Sampling error (%) is estimated as:

$$(100)\sqrt{\left[\frac{(S\bar{x} \text{ for plots})}{(\text{average plot volume})}\right]^2 + \left[\frac{(S\bar{x} \text{ for ratios})}{(\text{average ratio})}\right]^2} \qquad (8)$$

where:

$$S\bar{x} = \sqrt{\frac{\Sigma X^2 - (\Sigma X)^2/n}{(n)\ (n-1)}}$$

with:

$S\bar{x}$ = standard error

Σ = sum

X = individual observation, value, or ratio

n = number of samples

therefore:

$$S\bar{x} \text{ for plots} = \sqrt{\frac{(1)^2 + (3)^2 + \ldots (6)^2 - [(34)^2/10]}{(10)(10\text{-}1)}}$$

$$= 0.87$$

$$S\bar{x} \text{ for ratios} = \sqrt{\frac{(2.0000)^2 + (1.2000)^2 + (1.1667)^2 - \left[\frac{(4.3667)^2}{3}\right]}{(3)\ (3-1)}}$$

$$= 0.27$$

Estimated sampling error, then, is:

$$\text{Sampling error} = (100)\sqrt{\left(\frac{0.87}{3.4}\right)^2 + \left(\frac{0.27}{1.45557}\right)^2}$$

$$= 32\%$$

We are now ready to proceed to 3P sampling.

3P SAMPLING

In most forest inventory work, a list of the population of interest is not available prior to sampling. The 3P system is essentially a means of using some rough estimates of total sample size to develop a list of random numbers which will permit sampling with a probability proportional to predictions. The following explanation of 3P sampling is presented in easy-to-follow outline form.

1. Let us suppose we have a "forest" of ten trees we wish to inventory using 3P sampling. Although unknown to us, the actual volumes of the trees are:

Tree No.	Actual volume (board feet)
1	100
2	50
3	150
4	90
5	120
6	30
7	10
8	160
9	100
10	10
Total	820

2. To use 3P sampling we must determine the following:
   (1) The number of 3P sample trees needed. This can be done statistically using formula 3.

We will assume a coefficient of variation of the ratios of measured to estimated volumes of 20 percent and determine the number of 3P samples needed for a sampling error of 10 percent, 2 in 3 times.

$$n = \frac{(1)^2\,(20)^2}{(10)^2} = 4$$[1]

(2) Estimate the total volume, which we will assume is 1000 board feet.

(3) Estimate the total number of trees, which we will assume is 12.

(4) Estimate the volume of the largest tree, which we will assume is 150 board feet. All trees with an estimated volume of more than 150 board feet will be measured and will not be part of the 3P sampling process.

3. To select our 3P sample trees, we need a list of random numbers from 1 to "KZ."

(1) KZ = estimated total volume/desired number of 3P samples (9)
= 1000/4 = 250

(This indicates we will select one 3P sample tree for about every 250 board feet of volume.)

(2) Using a table of random numbers, we select 12 numbers from 1 through 250.

(3) Trees whose estimated volume is equal to or greater than a paired random number are 3P sample trees.

(4) Since any trees with estimated volumes greater than 150 board feet must be measured (termed "sure-to-be-measured"), random numbers larger than 150 will never result in the selection of a 3P sample. Therefore, we term these numbers "nulls" and designate them by the symbol "#." These nulls, of course, result in the automatic rejection of the tree as a sample, simplifying the use of 3P random numbers. If desired, this approach can be used to assure the measurement of all the largest and most valuable trees also.

---

[1] Loetsch *et al.* (1973) recommend that the probability of obtaining a smaller number of 3P samples be reduced by increasing n by the amount of $\sqrt{n - \frac{n^2}{N}}$, where N = total number of trees expected. In this example, $\sqrt{4 - \frac{(4)^2}{12}} = 1.6$, and n would become 4 + 1.6 = 5.6 or ≈6.

4. The random numbers and our estimated volumes are as follows:

Random number	Tree No.	Estimated volume
#	1	90
52	2	40
#	3	200
63	4	90
#	5	100
#	6	40
#	7	20
#	8	150
82	9	90
75	10	20
#		
#		

(1) You will note we generated more random numbers than were needed, which is no problem.

(2) Tree No. 3 must be measured as its estimated volume exceeds 150 board feet, and it is not part of our 3P estimate.

(3) The 3P sample trees, those whose estimated volume equals or exceeds their paired random number, are Nos. 4 and 9.

5. Our field sheet might appear as follows:

	Measured volume			
Tree No.	3P	100 percent	Estimated volume	Meas./Est.
1			90	
2			40	
3		160[a]		
4	90		90	1.0000
5			100	
6			40	
7			20	
8			150	
9	100		90	1.1111
10			20	
Total		160	640	2.1111

[a] This tree's estimated volume exceeded 150, thus it had to be measured and its volume is kept separate.

6. The average of the measured/estimated ratios is obtained:
   2.1111/2 = 1.0556
7. Our volume estimated[2] is:

   Volume = (average ratio) (total estimated volume) + (sure-to-be-measured)
   = (1.0556) (640) + 160 = 836 bd. ft.

   (close to the 820 "actual volume")
8. Using formula 2:

$$S = \sqrt{\frac{(1.0000)^2 + (1.1111)^2 - [(2.1111)^2/2]}{2 - 1}}$$

$$= 0.07856$$

From formula 1:

$$CV = \frac{0.07856}{1.0556}(100)$$

$$= 7.4\%$$

Sampling error, by formula 5, is:

$$\text{Sampling error} = 7.4/\sqrt{2}$$

$$= 5.2\%$$

We will now describe one approach to using 3P sampling in a timber marking job.

---

[2] As Mesavage (1971) points out, the blow-up factor (F = frequency) for a sample tree is obtained by the relation:

$$F = \frac{\text{sum of predicted volumes}}{(\text{number of 3P samples})(\text{predicted volume for given 3P sample tree})}$$

For tree 4, $F = \frac{640}{(2)(90)} = 3.556.$ As this tree's measured volume was 90, its contribution to the total estimate is (90) (3.556) = 320. For tree 9, $F = \frac{640}{(2)(90)} = 3.556.$ With a measured volume of 100, it contributes (100) (3.556) = 356. The sum of those contributions, 320 + 356 = 676, plus the sure-to-be-measured volume (160) gives a total volume estimate of 836 board feet as obtained above.

3P TIMBER MARKING USING A SPIEGEL RELASKOP AND HEIGHT ACCUMULATION

The 3P sampling system developed by L. R. Grosenbaugh selects trees for measurements with probability proportional to cheaply obtained guessed volumes or closely related values. Regardless of the size of the population, measurements on only about 100 trees are usually required for a statistical sampling error (2 times in 3) of plus or minus 2 percent (Space 1973). The volumes of the 3P sample trees can be obtained using standard volume tables, but this introduces whatever bias is inherent in those tables. Among the options to eliminate this problem are:

1. Measuring sample trees with a sophisticated dendrometer to determine volumes (Grosenbaugh 1974, Mesavage 1971).
2. Felling, bucking, and scaling sample trees to measure volumes (Johnson and Hartman 1972).
3. Measuring sample trees with a Spiegel Relaskop and using the height accumulation method of volume determination.

Felling, bucking, and scaling may not be physically convenient or economically feasible. Dendrometry often involves instruments costing several thousand dollars and complicated computer programs, also limiting its use. However, a method is described here utilizing the relatively inexpensive Spiegel Relaskop, in conjunction with the height accumulation method of volume determinations developed by Grosenbaugh (1954).[3] This provides a very practical combination. Yocom and Bower (1975) found volumes determined by this method were comparable to those obtained with Barr and Stroud dendrometers.

The Height Accumulation Method

Measurements needed for the height accumulation method include diameters outside bark (DOB) at various points of known height on the bole and diameter inside bark (DIB) at breast height (4.5 feet). The DIB/DOB ratio is calculated for diameters at breast height (0.90 in Figure 1). Diameter outside bark at breast height is rounded to the nearest even number of inches (14 in Figure 1), and then height, to the merchantable height limit, is successively measured in nearest number of 4-foot sections to each of the points on the bole where DOB is an even number of inches (2-inch taper steps).

---

[3] Grosenbaugh explains how trees may be graded using this system also, but that aspect will not be discussed in this bulletin.

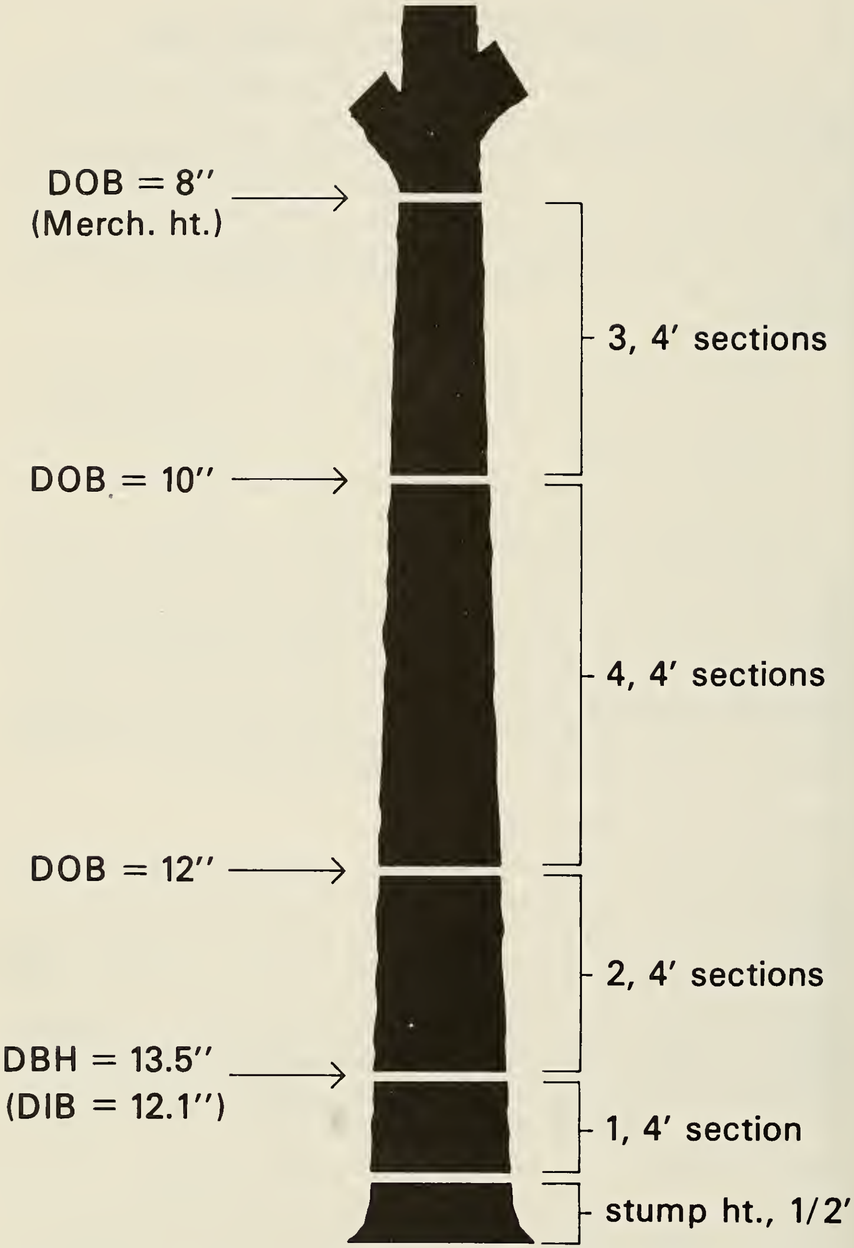

Figure 1. Measurements for determining volume by the height accumulation method.

To calculate volume from these measurements, we require the quantities L, H, and H′ corresponding to DOB at each 2-inch taper step (Table 2), where these are defined as:

L = number of 4-foot unit lengths between points where diameter is next lower DOB and given DOB (recorded as zero for 2-inch taper steps above merchantable heights); one 4-foot section is assumed from stump height to DBH.

H = height in number of 4-foot units (accumulated L-values) to given DOB's (value at merchantable height point repeated for remaining taper steps).

H′ = accumulated H-values.

We then calculate volume from these data and the relationship:

$$\text{Volume} = A(\Sigma H') + B(\Sigma H) + C(\Sigma L)$$

where the sum, Σ, is overall sections of the tree to a 2-inch top and the coefficients (A, B, and C) are from Table 3. For a DIB/DOB ratio of 0.90, for example, and data in Table 2, volumes are:

Cubic feet = (.141) (160) + (0) (51) + .0236 (10) = 23

Board feet (Int. 1/4″) = 1.29(160)-1.34(51)-.284(10) = 135

Board feet (Scribner) = 1.28(160)-1.61(51)-1.04(10) = 112

Board feet (Doyle) = 1.62(160)-4.50(51)+4.19(10) = 72

One might choose to use the coefficients appropriate for the mean DIB/DOB ratio for all the sample trees, with linear interpolations between the coefficients given in Table 3.

The Spiegel Relaskop is convenient for obtaining height accumulation data for 3P sample tree. The use of the instrument is outlined in **Appendix** B.

3P Procedure

As each tree is marked the cruiser makes an estimate of some quantity highly correlated with its volume. In view of the strong relation between cylindrical volume [BH = (basal area in square feet) (height in feet)] and volume as measured in any units, estimates of BH are recommended. Table 4 provides BH-values for field use. With experience, reasonable estimates of BH-values may be made without consulting the table.

Table 2. Height accumulation data for the tree shown in Figure 1.

	DOB, even inches							Sums, Σ
	14	12	10	8	6	4	2	
L	1	2	4	3	0	0	0	10
H	1	3	7	10	10	10	10	51
H'	1	4	11	21	31	41	51	160

Before beginning field work, one needs rough estimates of the following:

1. Total number of marked trees expected.
2. Expected sum of BH-values. (Estimate BH of what you assume might be an average tree for the sale and multiply by the number of marked trees expected.)
3. Anticipated BH of largest tree to be marked.

These estimates are used to obtain random numbers from 1 to "KZ," where KZ = (expected sum of BH-values)/(desired number of 3P sample trees). A random number is obtained for each tree to be marked. Any mumber from 1 through the KZ-value is used, although those above the BH of the largest tree expected can be designated as "nulls" since no trees paired with those numbers will be 3P samples. However, if a tree is encountered whose estimated BH exceeds the maximum expected, it must be measured (termed a "sure-to-be-measured tree"). These random numbers could be laboriously listed from a random numbers table, but they can be generated rapidly using a computer.

Starting with the first tree, field procedures are:

1. Estimate the BH of the tree to be marked, and record that estimate under the correct species column on a form such as that shown in Table 5, unless it exceeds the maximum BH expected.
2. If the estimated BH exceeds the maximum BH expected for a tree, collect height accumulation data as indicated in Table 6 and record under "Sure-to-be-measured trees." Go to the next tree.
3. If the estimated BH does not exceed the maximum expected, check your first random number.
4. If the estimated BH equals or exceeds the random number, list the estimated BH under the "3P sample trees" portion of Table 5 as sample tree number 1. Collect height accumulation data as indicated in Table 6. Go to the next tree.
5. If estimated BH is less than the random number or the random number is a null, go to the next tree.
6. Repeat this procedure for all trees marked.

Table 3.

Coefficients used in calculating volumes by the height accumulation method using a unit height of 4 feet and taper steps of 2 inches (from Grosenbaugh 1954).

DIB/DOB	Coefficients	Cubic feet	Int. ¼"	Scribner*	Doyle*
1.00	A	.175	1.59	1.58	2.00
	B	0	−1.48	−1.78	−5.00
	C	.0291	− .308	−1.08	4.00
.95	A	.158	1.44	1.43	1.80
	B	0	−1.41	−1.69	−4.75
	C	.0263	− .296	−1.06	4.09
.90	A	.141	1.29	1.28	1.62
	B	0	−1.34	−1.61	−4.50
	C	.0236	− .284	−1.04	4.19
.85	A	.126	1.15	1.14	1.44
	B	0	−1.26	−1.52	−4.25
	C	.0210	− .270	−1.02	4.28

*Coefficients for Scribner and Doyle rules are correct only on a 16-foot log basis.

Table 4. BH table.

DBH (in.)	Height (16-foot sections)									
	½	1	1½	2	2½	3	3½	4	4½	5
4	1	1	2	3	3	4	5	6	6	7
6	2	3	5	6	8	9	11	13	14	16
8	3	6	8	11	14	17	20	22	25	28
10	4	9	13	17	22	26	31	35	39	44
12	6	13	19	25	31	38	44	50	57	63
14	9	17	26	34	43	51	60	68	77	86
16	11	22	34	45	56	67	78	89	101	112
18	14	28	42	57	71	85	99	113	127	141
20	17	35	52	70	87	105	122	140	157	175
22	21	42	63	84	106	127	148	169	190	211
24	25	50	75	101	126	151	176	201	226	251
26	29	59	88	118	147	177	206	236	265	295
28	34	68	103	137	171	205	239	274	308	342
30	39	79	118	157	196	236	275	314	353	393
32	45	89	134	179	223	268	313	357	402	447
34	50	101	151	202	252	303	353	404	454	504
36	57	113	170	226	283	339	396	452	509	565
38	63	126	189	252	315	378	441	504	567	630
40	70	140	209	279	349	419	489	558	628	698

Table 5. 3P sampling form.

Est. BH by species			3P sample trees			
Y.P.	Beech	Totals	No.	Meas. vol.	Est. BH	Meas. vol./Est. BH
34	57		1	188	34	5.5294
42	70		2	380	70	5.4286
22						
					Avg. =	5.4790
98	127	225 ◄	Total BH			
537	696	1233 ◄	Volume	(ΣBHx5.4790)		

Table 6. Height accumulation data form.

					4′ lengths to DOB′s																		
3P	Est.	At 4.5′																					Volume
sample	BH	DOB	DIB	DIB/DOB	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	ΣL	ΣH	ΣH′	(Int. ¼″)
1	34	14.9	13.5	0.91									1	6	2	0	0	0	0	9	53	184	188
2	70	19.2	19.1	0.99						1	2	3	3	0	0	0	0	0	0	9	73	337	380
			Avg.	0.95																			
Sure-to-be measured trees																							
Y. P.		26.0	24.7	0.95			1	5	3	4	0	0	0	0	0	0	0	0	0	13	146	899	1085

Note: It may be more convenient to record actual heights in feet in the field, converting to number of 4-foot units later. Use the odd-add rule when a taper step bisects a 4-foot unit, and record a terminal length in excess of 2 feet as an additional 4-foot unit (Enghardt and Derr 1963).

Office procedures are:

1. Calculate the ratio of the volume determined from height accumulation data to estimated BH for each measured sample tree.
2. Calculate the average ratio for sample trees.
3. Multiply the total estimated BH for all trees marked, except "sure-to-be-measured trees," by the average ratio to obtain their volume.
4. The sum of this volume and the total volume for "sure-to-be-measured trees" gives the final volume estimate.
5. Calculate the coefficient of variation for the ratios in the sample using formula 1.
6. Calculate the sampling error as a percent (formula 5).

As an illustration, calculations from the simple example shown in tables 5 and 6 are:

1. Total volume = 225(5.4790) + 1085 = 2318
2. CV = (0.07128/5.4790) (100) = 1.3%
3. Sampling error = $1.3/\sqrt{2} = 0.9\%$

Our next section will concern the use of 3P sampling in point sample cruises.

CRUISING APPALACHIAN HARDWOODS WITH 3P AND POINT SAMPLING

Much time can be saved in the field by combining 3P and point sampling. Before the cruise, determine:

(1) the number of point samples desired. This can be done statistically (formula 3), but for a 10 percent accuracy 2 in 3 times in most second-growth Appalachian stands, about 100 are needed (BAF = 10).

(2) the number of 3P sample points needed for the desired accuracy. This can be determined statistically, but 20 should suffice in most cases.[4] For a statistical determination, the formula is:

$$n_2 = \frac{(CV_2)^2}{E^2 - (CV_1)^2/n_1} \qquad (10)$$

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[4] Dr. George M. Furnival points out that bias may be introduced by 3P sampling a high proportion of the first-stage point samples (personal communications).

where: n_2 = number of 3P samples

CV_2 = coefficient of variation for ratios of measured to estimated values

E = standard error desired expressed as a percent of the mean (must be set at a value somewhat greater than the error for a point sample cruise)

CV_1 = coefficient of variation for point samples

n_1 = number of point samples

(3) the approximate number of 16-foot logs expected on "in-trees" at the point samples ($\approx\Sigma KPI$). For example, if you assume point samples will average 3, 2-log trees, and you plan to have 100 point samples:

$$\approx\Sigma KPI = (6)\ (100) = 600$$

Develop a list of random numbers, one for each point sample, from 1 through KZ, where:

$$KZ = \frac{\approx\Sigma KPI}{\text{number of 3P samples desired}} \qquad (11)$$

Therefore, KZ = 600/20 = 30.

In the field, record by species the number of 16-foot logs, to the nearest 1/2-log, for each "in-tree" on a point sample. Then, total the number of logs on that point sample.

If that total is less than the random number for that point sample, go to the next point. If that total equals or exceeds the random number for that point, measure "in-trees" for per-acre volume determinations as normally done on point sample cruises. The Spiegel Relaskop and height accumulation method may be used as described earlier or tables, such as tables 7 and 8, are convenient, for example.

Table 7. Volume per acre per "in-tree" on one point sample (BAF=10), Int.-¼", form class 78.

	Number 16-foot logs								
DBH	1	1½	2	2½	3	3½	4	4½	5
10	660	880	1082	1210	1339				
11	697	924	1151	1303	1454				
12	713	943	1172	1350	1529	1631	1745		
13	727	976	1215	1410	1595	1714	1823		
14	729	982	1234	1431	1627	1748	1870		
15	750	1011	1271	1483	1695	1834	1972		
16	759	1024	1289	1504	1726	1883	2041		
17	767	1040	1306	1534	1763	1927	2092		
18	770	1041	1319	1551	1777	1947	2117		
19	782	1062	1341	1580	1819	1991	2169		
20	783	1072	1356	1594	1837	2015	2198	2340	2482
21	795	1090	1381	1627	1872	2063	2255	2409	2563
22	800	1099	1395	1645	1895	2092	2285	2452	2619
23	802	1103	1402	1659	1915	2110	2301	2478	2658
24	798	1100	1402	1663	1924	2112	2299	2487	2671
25	806	1113	1418	1682	1948	2145	2344	2534	2725
26	810	1122	1431	1696	1965	2171	2377	2572	2767
27	814	1129	1441	1714	1986	2192	2399	2601	2800
28	812	1128	1441	1715	1989	2195	2403	2607	2810
29	818	1136	1454	1731	2006	2215	2424	2638	2851
30	822	1142	1465	1742	2022	2232	2444	2664	2887

Table 8. Volume per acre per "in-tree" on one point sample (BAF=10), Doyle, form class 78.

	Number 16-foot logs								
DBH	1	1½	2	2½	3	3½	4	4½	5
10	257	312	367	385	403				
11	333	409	485	530	576				
12	369	459	548	612	675	688	713		
13	412	521	640	716	792	825	868		
14	449	580	701	785	870	916	963		
15	489	636	782	880	986	1043	1108		
16	516	673	831	945	1067	1146	1217		
17	545	716	888	1021	1154	1243	1325		
18	566	747	928	1075	1217	1313	1404		
19	599	792	986	1143	1300	1402	1509		
20	618	824	1030	1195	1360	1475	1585	1667	1754
21	641	861	1082	1256	1431	1556	1681	1780	1830
22	659	887	1118	1304	1486	1618	1751	1865	1975
23	677	916	1152	1346	1541	1676	1811	1936	2061
24	687	932	1177	1377	1577	1714	1851	1988	2124
25	706	961	1213	1424	1635	1784	1934	2077	2221
26	721	981	1244	1461	1677	1837	1997	2149	2301
27	736	1003	1273	1497	1724	1887	2051	2210	2369
28	742	1016	1289	1521	1755	1919	2083	2249	2415
29	754	1036	1317	1557	1796	1966	2136	2313	2490
30	767	1055	1342	1587	1832	2007	2181	2366	2552

After field work is completed, calculate:

(1) the ratio of measured per-acre volume (Y) at each 3P point sample to the estimated total number of logs at that point sample, or Y/KPI.

(2) the total per-acre volume estimates, which equal the average number of logs recorded for "in-trees" on all point samples times the average Y/KPI-ratio, or:

$$\text{Volume} = \left[\frac{\Sigma \text{KPI}}{\text{number of point samples}}\right]\left[\frac{\Sigma (\text{Y/KPI})}{\text{number of 3P samples}}\right] \qquad (12)$$

(3) the approximate sampling error, which includes that due to the point samples and that related to 3P, plus a covariance term which will be ignored in this paper as is usually done in practice (see formula 8).

The following is an unrealistically small sample for illustration:

Point sample	Random No.	ΣKPI			Y	Y/KPI
		Species A	Species B	Total		
1	5	1	1	2	–	–
2	8	4	1	5	–	–
3	3	2	2	4	2000	500
4	12	0	1	1	–	–
5	1	3	0	3	1800	600
	Total	10	5	15		1100
	Average	2	1	3		550

$$\text{Total volume} = \left(\frac{15}{5}\right)\left(\frac{1100}{2}\right) = (3)\ (550) = 1650$$

$$\text{Species A volume} = \left(\frac{10}{5}\right)\left(\frac{1100}{2}\right) = (2)\ (550) = 1100$$

$$\text{Special B volume} = \left(\frac{5}{5}\right)\left(\frac{1100}{2}\right) = (1)\ (550) = 550$$

$$S_{\bar{x}} \text{ for point samples} = \sqrt{\frac{(2)^2 + (5)^2 + \ldots (3)^2 - [(15)^2/5]}{(5)\ (5 - 1)}}$$

$$= 0.71$$

$$S_{\bar{x}} \text{ for 3P samples} = \sqrt{\frac{(500)^2 + (600)^2 - [(1100)^2/2]}{(2)(2-1)}}$$

$$= 50$$

$$\text{Sampling error (\%)} = (100)\sqrt{\left(\frac{0.71}{3}\right)^2 + \left(\frac{50}{550}\right)^2}$$

$$= 25\%$$

If one wished to include pulpwood in the cruise, a similar scheme, using number of bolts in "in-trees" and a separate list of random numbers, would be required.

Our last example of the application of 3P sampling will deal with updating a continuous forest inventory (CFI) system.

UPDATING A CFI SYSTEM USING 3P SAMPLING

In 1967 a series of 384 permanently marked point samples (BAF=10) was established 8 chains apart on cruise lines 10 chains apart on a 3500-acre portion of the West Virginia University Forest near Morgantown. This Appalachian hardwood forest, as are most in the state, is essentially evenaged with scattered residuals from past logging operations. It was about 36 years old in 1967.

A previous simulation indicated the potential of updating this CFI system using 3P sampling (Wiant 1975b). A field test of this updating scheme was made in 1975.

Procedure

To be consistent with previous inventories of this forest, sawtimber volumes (Int.-1/4") for trees 10 inches DBH or greater were determined using the form class 78 table prepared by Mesavage and Girard (1946) and cubic-foot volumes for all trees 4 inches DBH or greater were estimated using local volume tables.[5] The 1967 inventory indicated an average per-acre volume of 2883 board feet (±5%) and 1806 cubic feet (±2%).

---

[5] Local volume tables for this forest prepared by Professor Emeritus Allen W. Goodspeed.

Using the per-acre, board-foot volume estimates of the 1967 cruise as first-stage estimates (KPI) and re-measured point samples as second-stage, 3P samples (Y), the number of 3P samples needed to estimate sawtimber volume within 6 percent was calculated assuming a CV_2 of 20% (formula 10):

$$n_2 = \frac{(20)^2}{(6)^2 - (99)^2/384}$$

$$= 38$$

Random numbers were generated from 1 through KZ, where $KZ = \Sigma KPI/n_2 = 1107198/38 = 29137$. Those first-stage, per-acre estimates on which board-foot volumes equaled or exceeded paired random numbers were selected as 3P samples, 39 qualifying.

Results

For various reasons, only 29 of the 39 second-stage samples could be located on the ground. Average Y/KPI-ratios were 1.52760 for board-foot and 1.17321 for cubic-foot volumes. Estimates of per-acre volumes in 1975 were:

(1.52760) (2883) = 4404 board feet
(1.17321) (1806) = 2119 cubic feet

Yearly per-acre growth, then, is estimated as 190 board feet or 39 cubic feet.

Sampling errors were calculated as 7 percent for board-foot and 4 percent for cubic-foot estimates. The covariance terms (see Wiant 1975a) were ignored as Van Hooser (1972, 1973) had done in previous updating studies. Those terms were unreasonably large and negative in our case, indicating, perhaps, the need for the development of more appropriate formulae for error estimates as pointed out by Schreuder *et al.* (1968).

Discussion

Volume estimates were obtained re-measuring 29 of 384 point samples with sampling errors only 2 percent greater for board-foot and cubic-foot volumes than those found eight years earlier. The time-savings of this approach is obvious to those involved with expensive CFI systems.

Little cutting has occurred on the West Virginia University Forest. Cutting would introduce more variation between first- and second-stage samples, increasing the number of 3P samples needed for a specified sampling error.

CONCLUSIONS

This publication has attempted to illustrate the basic concepts of 3P sampling and to point out a few applications. Many other uses are made, such as in sampling the amount of woody debris on cutover areas, wildlife and range inventories, and log scaling. Undoubtedly many more applications will be developed. A recent annotated bibliography of 3P sampling by Lund (1975) should be consulted by those seeking more information.

LITERATURE CITED

Enghardt, H., and H. J. Derr. 1963. Height accumulation for rapid estimates of cubic volume. J. For. 61: 134-137.

Grosenbaugh, L. R. 1954. New tree-measurement concepts: height accumulation, giant tree, taper and shape. USDA Southern For. Exp. Sta. Occ. Paper 134.

Grosenbaugh, L. R. 1974. STX 3-3-73: tree content and value estimation using various sample designs, dendrometry methods, and V-S-L conversion coefficients. USDA Forest Service Res. Paper SE-117.

Johnson, F. A., and G. B. Hartman, Jr. 1972. Fall, buck, and scale cruising. J. For. 70:566-568.

Loetsch, F., F. Zohrer, and K. E. Haller. 1973. Forest inventory, Vol. II. BLV Verlagsgesellschaft, Munchen, West Germany.

Lund, H. G. 1975. 3P sampling: an annotated bibliography. USDA Forest Service, State and Private Forestry, Upper Darby, Pa.

Mesavage, C. 1971. STX timber estimating with 3P sampling and dendrometry. USDA Agri. Handbook 415.

Mesavage, C., and J. W. Girard. 1946. Tables for estimating board-foot volume of timber. USDA Forest Service, Wash., D. C.

Schreuder, H. T., J. Sedransk, and K. D. Ware. 1968. 3-P sampling and some alternatives, I. For. Sci. 14:429-453.

Space, J. C. 1973. Three-P cruise your next timber sale. The Consultant 18:64-68.

Van Hooser, D. D. 1972. Evaluation of two-stage 3P sampling for forest surveys. USDA Forest Service Res. Paper SO-77.

Van Hooser, D. D. 1973. Field evaluation of two-stage 3P sampling. USDA Forest Service Res. Paper SO-86.

Wiant, H. V., Jr. 1974. Combine 3P and point sampling for efficient cruising. W. Va. Forestry Notes 2:12-15.

Wiant, H. V., Jr. 1975a. Influence of rounding on calculations of relative errors of combined 3P and point sample cruises. W.Va. Forestry Notes 4:7-8.

Wiant, H. V., Jr. 1975b. Updating a line-plot cruise on the West Virginia University Forest by 3P and point sampling. W.Va. Forestry Notes 4:9.

Yocom, H. A., and D. R. Bower. 1975. Estimating individual tree volumes with Spiegel Relaskop and Barr and Stroud dendrometers. J. For. 73:581-582,605.

APPENDIX

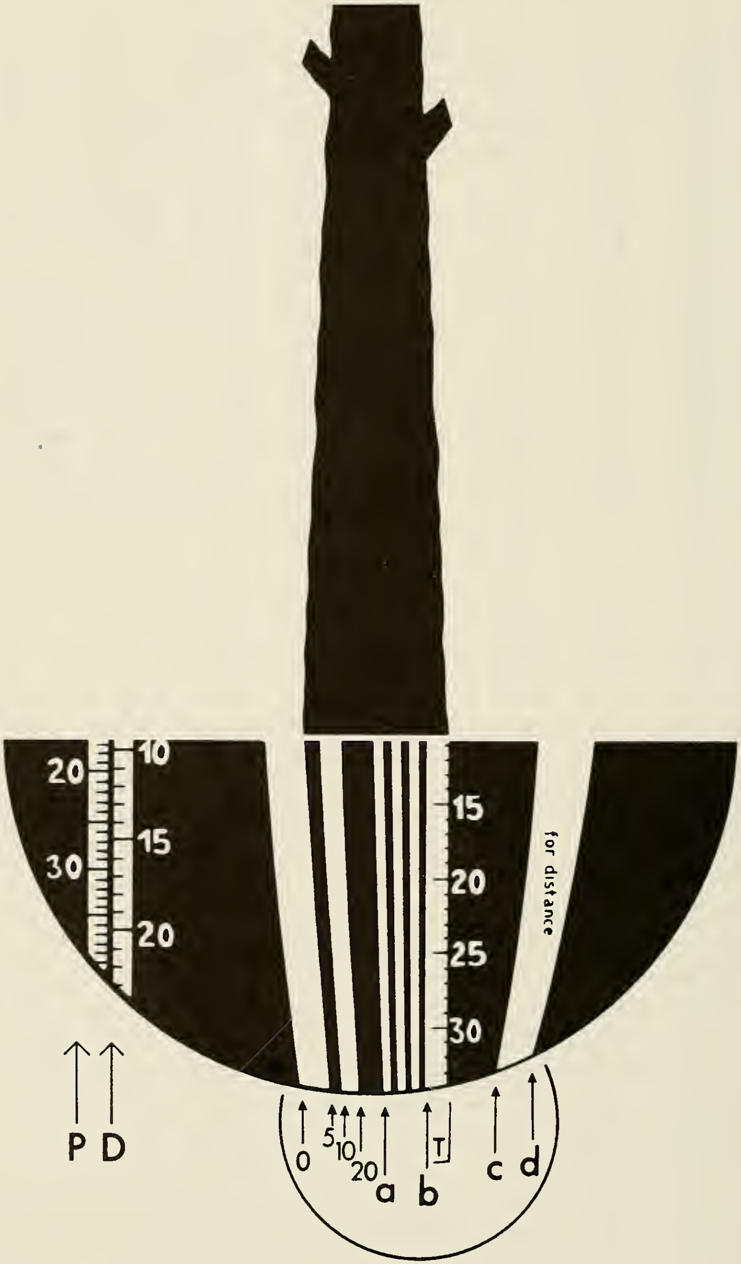
Appendix A. Table of random numbers.

1	2	3	4	5	6	7	8	9	10
58307	24257	81752	60907	72004	36419	07635	04538	60335	19790
90907	03450	70808	66317	88119	56155	21404	31615	49714	73898
36293	00045	53424	23207	55457	28486	20712	16831	41706	43052
52503	01298	49308	72993	08263	26094	00658	88112	09467	48514
22597	37628	29946	16800	39632	56733	19094	55239	74550	06918
55301	91220	68448	87206	49334	45270	53718	81933	61947	00612
03732	45379	84982	28984	30673	12818	57816	56045	96476	17751
08241	15982	64481	40541	89506	71733	76668	25318	97814	14789
38984	63475	69955	75695	95683	80244	23367	34229	36630	28888
30535	40780	28705	95142	74797	27081	77633	92647	61994	37762
85522	82625	32450	59107	69136	85337	51346	86759	97820	95097
80149	96124	41056	14112	04663	74761	89031	16018	36780	57831
24327	87583	90611	09238	35228	68060	81892	64531	66631	76804
05479	39913	27708	67915	21599	73734	74871	71398	32303	27327
58905	57473	03541	69544	26370	38272	71265	70542	36784	92477
08235	66721	02679	16777	64741	09897	66689	45721	66748	81847
38811	34906	77704	86520	77499	87023	33990	25909	35681	73570
25516	12272	53425	09079	47466	23672	85723	51356	34742	33532
39969	55901	49326	63302	76517	86488	85963	89322	07528	72425
59096	21124	30441	35768	18989	08143	92928	90329	18301	00329
13790	12605	82788	37267	50689	36144	94912	19542	30715	09539
99900	65545	00863	80746	69967	48165	52435	66708	90725	76641
97093	00814	25037	41630	29038	96787	20622	34536	31030	22587
15684	23596	26068	07278	42095	06827	98046	01547	99873	55021
54825	84290	55975	11048	20749	97989	43323	44855	96316	95604
89917	44410	23271	20387	01709	41676	56366	00708	93178	72508
07581	87885	74867	91216	49565	08590	34627	22846	02167	02731
19860	48671	71129	45258	37386	49098	04188	62528	62837	79204
75926	11458	62736	12470	84191	23837	21438	53081	22260	96920
01845	32296	19337	61634	41541	91268	21697	39353	45550	10690
53509	36597	60767	87387	04691	46740	29212	41229	20939	09996
51758	61315	62231	34220	36031	56048	47137	95653	07228	89879
00990	78136	04703	92382	44888	25388	66983	73928	09623	06480
28716	65948	36388	79080	01751	36255	42502	43914	79057	87916
32757	12479	55249	93312	50767	51403	32547	73496	92666	49576
49946	61887	02223	06037	72237	90684	43869	31396	87319	37714
48444	94716	64469	75072	94871	29838	72206	10479	32254	93715
04871	46778	69593	77076	86108	65290	93982	03894	35377	17726
41255	56548	18186	35198	97145	93412	25471	12918	25939	14054

Appendix A. Table of random numbers (cont.)

1	2	3	4	5	6	7	8	9	10
6382	39897	27393	20740	17214	08935	38660	74632	52221	07564
5080	57001	94385	01467	99193	59107	21126	64338	14413	19347
7325	53033	37159	42534	76594	14089	12657	65803	17970	61054
2420	37951	77609	33494	21223	08583	67054	08288	21129	70563
0176	00567	50668	71322	15471	48898	44559	40340	12736	46334
5101	16440	69363	68335	48667	18035	92218	69846	69353	43675
7924	76752	11535	81727	11346	23003	74318	25534	11250	66586
9798	98869	34511	70087	29029	67087	55208	40486	26264	31001
4150	67192	00816	32534	41852	45529	01026	74093	61644	99017
0355	48576	23676	43486	13712	20332	29743	48698	87683	71506
0307	08700	86608	61086	97655	89624	62536	12234	42795	73671
3903	52297	11643	71506	32000	99102	13535	54773	41056	36472
8199	16625	37647	73688	27986	73969	92514	88419	90630	57687
7782	82129	91755	36966	11601	45091	82915	64149	28269	72917

Appendix B. Use of the Spiegel Relaskop.



1. "P" is a percent scale.
2. "D" is a degree scale.
3. "0 to a" gives basal area factors (BAF 5, 10, or 20) with automatic slope correction. For example, a tree which at DBH appears larger than the "0 to 10" interval (broad white band plus narrow black band) is an "in-tree" for BAF 10.
4. "a to b" measures upper stem diameters, with each light or dark bar equal to:

   2" at 33'
   4" at 66'
   6" at 99'
   8" at 132'

   (distance "a to b" = distance "0 to 10" = distance "10 to a")
5. "t" gives height in feet when 66 feet from the tree, and double height when 33 feet away.
6. "b to d" is a rangefinder. Stand a 6-foot stick against a tree. From about 66 feet, sight at the center of the stick and fix slope by setting the brake. Turn Relaskop 90°, and move until "b to d" intercepts ends of the stick. You are now 66 horizontal feet from the tree. Use a 3-foot stick for 33 feet.